Bell's Inequalities, EPR, locality, and all that

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Bell's Theorem is a Big Deal!

From Mermin's Moon paper:
We read, for example, in The Dancing Wu Li Masters that:

Some physicists are convinced that [Bell's theorem] is the most important single work, perhaps, in the history of physics.

And indeed, Henry Stapp, a particle theorist at Berkeley, writes that: "Bell's theorem is the most profound discovery of science."

Quantum Mechanics Pre-EPR

- Orthodox Quantum Mechanics (not just quantum theory) holds that states of quantum systems do **not exist** until observed.
- Equivalently: There is no **physical reality** indendent of an observer.
- Einstein disagreed, and he and Niels Bohr debated this over several years, without resolution.
- Einstein (initially) thought that QM was incorrect, and later thought that it was correct, but incomplete.
- In 1935 Einstein, Podolsky and Rosen (EPR) published a paper that sharpened the argument for the incompleteness of QM.

The EPR Paper – Four Goals

1. To give a definition for "physical reality".
2. Describe an experiment where the definition of "physical reality" was met – so physical reality exists.
3. Argue that QM was incomplete because it does not include any concept of "physical reality".
4. Show that orthodox QM implies some weird (spooky) stuff, that had not been pointed out before. This was intended as a strong argument against the completeness of QM, because

   > *"No reasonable definition of reality could be expected to permit this" From EPR*

   But now we know that reality (or nature anyway) *does* permit this weird, spooky stuff – and we can exploit it.

EPR Definition of Physical Reality

From EPR "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

Modified EPR Thought Experiment

- Central source creates two particles with some property that has total value of zero, that must be conserved (e.g., total angular momentum?) and shoots one to the right, and one to the left. The value of the property (zero) must be conserved. So if one particle has value $+X$, the other must have value $-X$.
- Two (property) detectors - one on the right and one on the left of the source. The one on the right is *slightly* closer to the source then the one on the left. So, the right particle hits its detector before the left particle hits its detector.

Left Detector $< - - - - - - - -$ Source $- - - -- >$ Right Detector

Modified EPR Thought Experiment

- Suppose the right one detects the value "+X". So at that **instant**, before it is observed, and without additional disturbing the system, we can predict with certainty that when the left particle reaches its detector, its value will be "-X". I.E., the two values are totally (anti)-correlated.
- So that property meets the definition of a physical quantity with a physical reality. So, physically real quantities exist without being observed, and QM is incomplete.

And Most Important and Disturbing

- The right and left detectors are far enough apart that from the time that the right particle reaches its detector, to the time when the left particle hits its detector, there isn't enough time for light to travel from the right detector to the left detector.
- But **if**, as in orthodox QM, the property of the left (right) particle does not exist before it is observed, there must be some kind of faster-than-light "influence", or "spooky action" that occurs, to fix the value of the left particle (instantly), when the value of the right particle is determined.
- This "influence" is now called **Entanglement**.

Einstien Expected

- There should be a theory to explain the results of the thought experiment, without requiring such "influences", i.e., where the particles have real, fixed, properties at the time that they leave the source, and that determine what the detectors will observe.
- Such a theory is now called "local, realistic". *Local* meaning that there is no influence between the detectors faster than the speed of light, and *realistic* meaning that the properties exist without first needing to be observed.
- Also, a realistic theory is said to be a theory with *hidden variables*,
  "such that ... [the] values of these variables together with the state vector determine precisely the results of individual measurements." Bell 1964

The impact of EPR

For Bohr, the EPR paper "came out of the blue" and he dropped everything he was doing to respond. Few people claim to understand his response.
But, for the most part, physicists did not notice the EPR paper.

The question of whether results in the EPR experiment were due to entanglement, or due to an unknown local, realistic (hidden variables) theory, seemed only **philosophical**, with no practical consequence, and more important, **no experimental way** to resolve the question.

So, EPR was largely ignored for about thirty years.

Enter John Bell 1964

"I was a quantum mechanic during the week, but on Sundays I had principles." A misquote from John Bell

Von Neumann impossibility proof

- EPR was ignored partly due to a published "proof" by John Von Neumann that a local, realistic theory explaining the EPR experiment was *impossible*. There were also two other such "proofs".
- Bell realized that the Von Neumann argument should also apply to Bohm's QM theory. So, Bell saw that Bohm had "done the impossible".
- Bell found the error in Von Neumann's paper, the claim that:

$$E(X_1 + X_2) = E(X_1) + E(X_2),$$

  where X_1 and X_2 are observables, i.e., a Hermitian matrices.
- Of course this is true when each X_i is a random variable (linearity of expectations), but Von Neumann used it for states which have "no statistical character".

From Bell 1964

"His essential assumption is: Any real linear combination of any two Hermitian operators represents an observable, and the same linear combination of expectation values is the expectation value of the combination."

Bell's theorem(s)

- Several versions of Bell's theorem have been developed, that are used in the context of thought (and now real) experiments that are different, but related, to the modified EPR thought experiment. Many of the Bell theorems are in the form of inequalities.
- The goal of Bell theorems: To show that experiments are possible, where the experimental results *predicted* by QM cannot be explained (reproduced) by *any* local, realistic theory.
- But, how can we *model* the set of *all* local, realistic theories?
- Some of the proofs more *explicitly* (and to me more convincingly) establish a model, and more convincingly establish the **impossibility** of any local, realistic theory to explain the QM predictions.

Three proofs (expositions)

1. A very simple, somewhat physical, proof from the book "Quantum Enigma".
2. A more principled, and general impossibility argument (from D. Mermin).
3. A simple algebraic argument.
4. Zeilinger's flawed exposition.

Quantum Enigma, and Mermin Expositions

Here I moved to pages copied from those sources, in order to use their illustrations.
Note, in my lecture I did not do a good job of describing the argument from Quantum Enigma. So, I recommend people read those pages - the authors do a fairly good job.

Simple Algebraic Proof - On the internet, from David Harrison, U. Toronto Physics

Consider a set of elements, where each is described by three binary properties A, B, C. If you want to be concrete, imagine a set of people, and let A be the property of being male; B the property of being tall; and C the property of having blue eyes.

Claim: In any set of elements with any distribution of the three properties:

$$\#(A \wedge \overline{B}) + \#(B \wedge \overline{C}) \geq \#(A \wedge \overline{C}).$$

Do try this at home, or in a class of students.

The Proof

Certainly,

$$\#(A \wedge \overline{B} \wedge C) + \#(\overline{A} \wedge B \wedge \overline{C}) \geq 0,$$

since both terms must be non-negative.
Now add

$$\#(A \wedge \overline{B} \wedge \overline{C}) + \#(A \wedge B \wedge \overline{C})$$

to both sides. This yields

$$\#(A \wedge \overline{B} \wedge C) + \#(\overline{A} \wedge B \wedge \overline{C})$$

$$+\#(A \wedge \overline{B} \wedge \overline{C}) + \#(A \wedge B \wedge \overline{C})$$

$$\geq \#(A \wedge \overline{B} \wedge \overline{C}) + \#(A \wedge B \wedge \overline{C}) = \#(A \wedge \overline{C})$$

Similarly, the left side of the inequality cleans up to

$$\#(A \wedge \overline{B}) + \#(B \wedge \overline{C}).$$

So, $\#(A \wedge \overline{B}) + \#(B \wedge \overline{C}) \geq \#(A \wedge \overline{C})$.

Why is this inequality relevant?

The set of elements are electrons. Two identical electrons are shot in opposie directions into "Stern-Gurlach" detectors.
The three binary properties are:

1. A: spin up when the SG detector is upright (0 degrees)
2. B: spin up when the SG detector is diagonal (45 degrees)
3. C: spin up when the SG detector is horizontal (90 degrees)

In the experiment, we can't measure two of these properties with a single electron, so can't determine say $\#(A \wedge \overline{B})$. But, we can measure one property on one electron, and the other property on the other (identical) electron. And so, we can see if the above Bell inequality is violated in the experiment - it is!

Impossibility model?

This is an inequality that is violated in experiments - and is contrary to what QM calculations predict for those experiments. So, it can be used to show that quantum phenomina violate simple counting arithmetic (as in the Bell inequality), and that something non-classical (spooky) is going on. But it is not as clear how assumptions of locality and realism come into the argument.

Hence, it is not as clear (to me) exactly what “impossibility” is established from this inequality/experiment.

Impossibility??

Quote from Bell:

> *... what is proved by impossibility proofs is lack of imagination.*

I'm lost! How do we reconcile this quote with the fact that Bell is famous for proving impossibility? Maybe some spooky action?