ECS 20 **Homework #2, Proofs and Set Theory (52 points)** **Winter 2012**

Due: January 26th by 4:00pm
Each problem is worth 4 points.

From text: 1.31a, 1.38a, 1.38b, 1.52, 1.56. 1.59c

1. Given $A = \{1, 2, \ldots, 9\}$, $B = \{2, 4, 6, 8\}$, $C = \{1, 3, 5, 7, 9\}$, $D = \{3, 4, 5\}$, $E = \{3, 5\}$. Which of these sets can equal a set X under each of the following conditions.
a. X and E are disjoint.
b. $X \subseteq$ C but $X \not\subset D$
c. $X \cap E = E$
d. $X \cup A = A$

2. Given $\mathbf{U} = \{1, 2, \ldots, 9, 10\}$, $B = \{2, 4, 6, 8\}$, $C = \{1, 3, 5, 7, 9\}$, $D = \{3, 4, 5\}$, $E = \{3, 5\}$
a. Provide a Venn diagram to describe the sets.
b. Provide the power set $P((C^C \cup D)^C)$. Hint: check your answer using $n P((C^C \cup D)^C)$
c. Is $[\{2, 3, 4, 6\}, \{4, 5\}, \{8\}]$ a partition of $B \cup D$? Why or why not?
d. What is $nP(B \cup E)$?

3. Given $A \cap B \neq \varnothing$, $B \cap C \neq \varnothing$, $A \cap C = \varnothing$, $A \not\subset B$, $C \not\subset B$. Draw a Venn diagram to describe sets A, B, C.

4. Use contraposition to prove that if a product of two positive real numbers is greater than 100, then at least one of the numbers is greater than 10.

5. Any real number that is not rational is "irrational". Prove by contradiction that the square root of any irrational number is irrational.

6. Use a proof by cases to show that $|xy| = |x||y|$, where x and y are real numbers. Note that $|a|$ equals a when $a \geq 0$, and equals $-a$ when $a \leq 0$.

7. Use the rules of inference to show that if $\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$, and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.