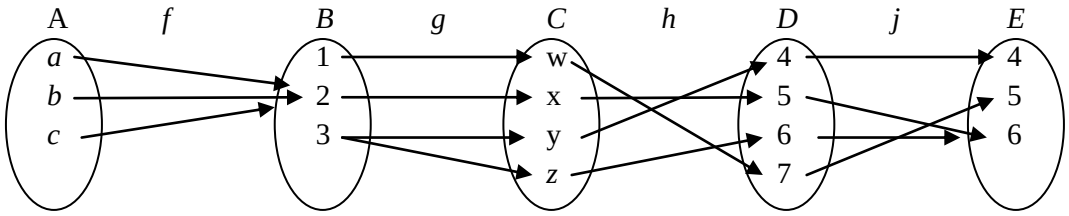
ECS 20 **Homework #3, Functions and Algorithms (34 points)** **Winter 2012**

Due: Wednesday, February 1st by 4:00pm

1. (5 points) Given $B = \{2, 4, 6, 8\}$, and $C = \{1, 3, 5, 7, 9\}$, and a relation f from B to C.
   a. If $f = \{(2, 3), (6, 1), (4, 3), (8, 7)\}$, is f a function? If not, then why not?
   b. If $f = \{(2, 5), (4, 1), (6, 7), (8, 9), (4, 3)\}$ is f a function? If not, then why not?
   c. If $f = \{(4, 3), (2, 9), (8, 4)\}$ is f a function? If not, then why not?
   d Could there be a function f that would be one-to-one? If it could, then provide such a relation. If not, then provide a reason such an f could not exist.
   e. Could there be a function f that would be onto? If it could, then provide such a relation. If not, then provide a reason such an f could not exist.

2. (4 points) Let the relations $f: A \to B$, $g: B \to C$, $h: C \to D$, $j: D \to E$ be defined by the figure below.



   Which relations are
   a) a function?
   b) onto?
   c) one-to-one?
   d) invertible?

3. (1 point) Let f: $\mathbf{R} \to \mathbf{R}^+$ be defined by $f(x) = 4x^2 + 12x + 9$, what is $f^{-1}(y)$?

4. Suppose f: $A \to B$ is one-to-one and $g: A \to B$ is onto. Let $x \subseteq A$.
   a. (2 points) Show that $f1_x$, the restriction of f to x, is one-to-one.
   b. (2 points) Show that $g1_x$, the restriction of g to x, need not be onto.

5. (7 points) Provide the value for each of the following standard functions
   a. $\lfloor -19.6 \rfloor$
   b. $\lceil -19.6 \rceil$
   c. INT(-19.6)
   d. INT(19.6)
   e. -19 mod 6
   f. $\log_7 (49)^4$
   g. $\frac{((n+1)!)^2}{(n!)^2}$

6. (4 points) Let F_i be the Fibonacci numbers, where $F_0 = 1$, $F_1 = 1$, $F_2 = 2$, $F_3 = 3$, $F_4 = 5$ …. Prove $\sum_{i=1}^{N-2} F_i = F_N - 2$, where $N > 2$

7. (2 points) What is the big-O of the following snippet of code?

```
for(i = 0; i < n; i = i + 1)
  for(j = 0; j < i; j = j + 1)
    count = count + 1;
```

8. (2 points) What is the big-O of the following power() function? (Hint: it is in terms of the exponent's value)

```
int power(int x, int exponent)
{
  if(exponent == 0)
    return 1;

  if(exponent == 1)
    return x;

  if(exponent mod 2 == 0) // exponent is even
    return power(x * x, exponent / 2);
  else
     return x * power(x * x, exponent / 2);
}
```

9. (5 points) Prove that $x^2 + 3x - 10$ is $\Theta(N^2)$. This actually involves two proofs.