Laws of the Land of ECS 20

Laws of Logical Equivalence

Name	Or version	And version
Commutative	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Associative	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributive	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Idempotent	$p \vee p \equiv p$	$p \wedge p \equiv p$
Identity	$p \vee \mathrm{F} \equiv p$	$p \wedge \mathrm{T} \equiv p$
	$p \vee \mathrm{T} \equiv \mathrm{T}$	$p \wedge \mathrm{F} \equiv \mathrm{F}$
Complement	$p \vee \neg p \equiv \mathrm{T}$	$p \wedge \neg p \equiv \mathrm{F}$
	$\neg \mathrm{T} \equiv \mathrm{F}$	$\neg \mathrm{F} \equiv \mathrm{T}$
Double Negative	$\neg(\neg p) \equiv p$	
De Morgan's	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$
	$\neg(\forall x \in A)p(x) \equiv (\exists x \in A)\, \neg p(x)$	
	$\neg(\exists x \in A)p(x) \equiv (\forall x \in A)\, \neg p(x)$	
Absorption	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$

Rules of Inference

Name	Rule of Inference	Tautology
Modus [ponendo] ponens = "the way that affirms by affirming"	p $\underline{p \to q}$ $\therefore q$	$(p \wedge (p \to q)) \to q$
Modus [tollendo] tollens = "the way that denies by denying"	$\neg q$ $\underline{p \to q}$ $\therefore \neg p$	$(\neg q \wedge (p \to q)) \to \neg p$
Transitivity (Hypothetical syllogism)	$p \to q$ $\underline{q \to r}$ $\therefore p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$
Elimination (Disjunctive syllogism)	$p \vee q$ $\underline{\neg p}$ $\therefore q$	$((p \vee q) \wedge \neg p) \to q$
Generalization	$\underline{p}$ $\therefore p \vee q$	$p \to (p \vee q)$
Simplification	$\underline{p \wedge q}$ $\therefore p$	$(p \wedge q) \to p$
Conjunction	p $\underline{q}$ $\therefore p \wedge q$	$((p) \wedge (q)) \to (p \wedge q)$
Resolution	$p \vee q$ $\underline{\neg p \vee r}$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to (q \vee r)$
Universal instantiation	$\underline{\forall x\, p(x)}$ $\therefore p(c)$	
Universal generalization	$\underline{p(c) \text{ for an arbitrary } c}$ $\therefore \forall x\, p(x)$	
Existential instantiation	$\underline{\exists x\, p(x)}$ $\therefore p(c)$	
Existential generalization	$\underline{p(c) \text{ for some element } c}$ $\therefore \exists x\, p(x)$	

Laws of the Algebra of Sets

Name	Union version	Intersection Dual version
Commutative	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Associative	$(A \cup B) \cup D = A \cup (B \cup D)$	$(A \cap B) \cap D = A \cap (B \cap D)$
Distributive	$A \cup (B \cap D) = (A \cup B) \cap (A \cup D)$	$A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$
Idempotent	$A \cup A = A$	$A \cap A = A$
Identity	$A \cup \varnothing = A$	$A \cap \mathbf{U} = A$
	$A \cup \mathbf{U} = \mathbf{U}$	$A \cap \varnothing = \varnothing$
Complement	$A \cup A^C = \mathbf{U}$	$A \cap A^C = \varnothing$
	$\mathbf{U}^C = \varnothing$	$\varnothing^C = \mathbf{U}$
Double Complement	$(A^C)^C = A$	
De Morgan's	$(A \cup B)^C = A^C \cap B^C$	$(A \cap B)^C = A^C \cup B^C$
Absorption	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Set Difference	$A - B = A \cap B^C$	