Due: Thursday, February 16th by 4:00pm

1. (5 points) Hexadecimal numbers are numbers using base 16, instead of the usual base 10, and are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. $A_{16} = 10$, $B_{16} = 11$, $C_{16} = 12$, $D_{16} = 13$, $E_{16} = 14$, $F_{16} = 15$, $10_{16} = 16$, $2B_{16} = 2 * 16 + 11 = 43$, $9A2D_{16} = 9 * 16^3 + 10 * 16^2 + 2 * 16 + 13 = 39469$.

a) How many 4-digit hexadecimal numbers are there?

b) How many 4-digit hexadecimal numbers begin with one of the digits 3 through B, and end with one of the 5 through F?

c) How many 4-digit hexadecimal numbers are greater than $BFFF_{16}$?

d) A given random number generates each 4-digit hexadecimal number with equal probability. What is the probability that it generates a 4-digit hexadecimal number greater than $BFFF_{16}$?

e) What is the probability that the random number generator produces a number that begins with one of the digits 3 through B, and ends with one of the 5 through F?

2. (5 points) Urn #1 contains two black balls (labeled B_1 and B_2), and one white ball. Urn #2 contains one black ball and two white balls (labeled W_1 and W_2).

a) If I randomly choose an urn, and then select two balls from that urn, either construct a tree of the different outcomes as covered in section 5.8 in the text, or list them?

b) If I randomly choose an urn, and then select two balls from that urn, what is the size of the event space that the balls are the same color?

c) If I randomly choose an urn, and then select two balls from that urn, what is the probability as a simplified fraction that balls are different colors?

d) If I choose one ball from Urn #1, and then choose one from Urn #2, what is the size of the sample space?

e) I choose one ball from Urn #1, and then choose one from Urn #2, what is the probability as a simplified fraction that the balls are the same color?

3. (5 points) Answer the following questions about standard California license plates. They start with a digit, 1-9, followed by three letters, and finally have 3 digits, 0-9. The police found three people at the scene of crime that said they had seen the criminals drive off in a car.

a) How many different license plates are there?

b) The first witness said they saw part of a license plate, it had an 'A', a 'J', a 7, and '49', how many license plates could match that description.

c) The second witness said the license plate was 5F?C 383, but the F could be an E or R, and she really has troubles differentiating between 3's and 8's. How many license plates could match that description?

d) The last witness said that the license plate was composed of vowels (a, e, i, o, u, and y) and even digits, but each appeared no more than once. How many license plates could match that description?

e) Upon further investigation, the police discovered that all three witnesses had been paid by the criminals to mislead the police by giving a description of what the license plate was not. From witness b), they figured that there is no 'A', 'J', '7', nor '49'. Based on witness c)'s description, the police now know that the plate does not start with a 5, nor is the first letter an F, nor is the third letter a C, and the 3-digit numbers do not contain the digits 3 and 8. From witness d), they now understand that the license contain only consonants, uses only odd digits, and things may be repeated. How many license plates could match this new description?

4. (5 points) Assume the birthdays are equally likely to occur in any one of the 12 months of the year.

a. Given a group of four people, A, B, C, and D, what is the total number of ways in which birth months could be associated with A, B, C, and D? (For instance, A and B might have been born in May, C in September, and D in February. As another example, A might have been born in January, B in June, C in March, and D in October.)

b. How many ways could birth months be associated with A, B, C, and D so that no two people would share the same birth month?

c. How many ways could birth months be associated with A, B, C, and D so that at least two people would share the same birth month?

d. What is the probability that at least two people out of A, B, C, and D share the same birth month?

e. How large must n be so that in any group of n people the probability that two or more share the same birth month is at least 50%

5. (5 points) ECS 20 has 122 students. All of the students in the class are known to be from 17 through 34 years of age. You want to make a bet that the class contains at least x students of the same age. How large can you make x, and yet be sure to win your bet?

6. (5 points) Let A be a set of six positive integers each of which is less than 13. Show that there must be two distinct subsets of A whose elements when added up give the same sum. For example, if $A = \{1, 3, 4, 5, 10, 12\}$, then the elements of the subsets $\{1, 4, 10\}$ and $\{5, 10\}$ both add up to 15. Hint: What is the size of the power set of A?

7. (5 points) An interesting use of the inclusion/exclusion rule is to check survey numbers for consistency. For example, suppose a public opinion polltaker reports that out of a national sample of 1200 adults, 675 are married, 682 are from 20 to 30 years old, 684 are female, 195 are married and are from 20 to 30 years old, 467 are married females, 318 are females from 20 to 30 years old, and 165 are married females from 20 to 30 years old. Are the polltaker's figures consistent? Could they have occurred as a result of an actual sample survey?

8. (5 points) Find an explicit formula for the sequence $a_n = 4a_{n-2}$, for all integers $n \geq 2$ with $a_0 = 1$, $a_1 = -1$.

9. (5points) Find an explicit formula for the sequence $a_n = -4a_{n-1} + 4a_{n-2}$, for all integers $n \geq 2$ with $a_0 = 6$, $a_1 = 8$.

10. (5 points) Show that if r_1, r_2, a_0, a_1 are numbers with $r_1 \neq r_2$, then there exist unique numbers c_1 and c_2 so that $a_0 = c_1 + c_2$ and $a_1 = c_1r_1 + c_2r_2$.

11. (5 points, extra credit) Given a multiset of duplicated items with counts n_1, n_2, n_3 ... n_k, determine a formula for the number of permutations for r items selected from the multiset. For example, if the multiset contained $A = \{1, 2, 2, 3, 4, 4, 4\}$, the permutations for 3 items would be [$\{1, 2, 2\}$; $\{1, 2, 3\}$; $\{1, 2, 4\}$; $\{1, 3, 2\}$; $\{1, 3, 4\}$; $\{1, 4, 2\}$; $\{1, 4, 3\}$; $\{1, 4, 4\}$; $\{2, 1, 2\}$; $\{2, 1, 3\}$; $\{2, 1, 4\}$; $\{2, 2, 1\}$; $\{2, 2, 3\}$; $\{2, 2, 4\}$; $\{2, 3, 1\}$; $\{2, 3, 2\}$; $\{2, 3, 4\}$; $\{2, 4, 1\}$; $\{2, 4, 2\}$; $\{2, 4, 3\}$; $\{2, 4, 4\}$; $\{3, 1, 2\}$; $\{3, 1, 4\}$; $\{3, 2, 1\}$; $\{3, 2, 2\}$; $\{3, 2, 4\}$; $\{3, 4; 1\}$; $\{3, 4, 2\}$; $\{3, 4, 4\}$; $\{4, 1, 2\}$; $\{4, 1, 3\}$; $\{4, 1, 4\}$; $\{4, 2, 1\}$; $\{4, 2, 2\}$; $\{4, 2, 3\}$; $\{4, 2, 4\}$; $\{4, 3, 1\}$; $\{4, 3, 2\}$; $\{4, 3, 4\}$; $\{4, 4, 1\}$; $\{4, 4, 2\}$; $\{4, 4, 3\}$; $\{4, 4, 4\}$] = 43 ways.